PROPOSED ULTRA HIGH SPEED RELAYING SCHEME FOR MULTIPHASE SYSTEMS

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CERTIFICATE

This is to certify that the work entitled, 'PROPOSED ULTRA HIGH SPEED RELAYING SCHEME FOR MULTIPHASE SYSTEMS' by Amulya Chandra has been carried out under our supervision and has not been submitted elsewhere for award of a degree.

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CONTENTS

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LIST OF SYMBOLS

v,V	Voltage
i,I	Current
R	Resistance
Z	Impedance
Z_c	Surge impedance
Y	Admittance
S	Transform parameter
J	Current source
ϕ_o	Fault initiation angle
ω_o	Nominal system frequency
σ	Propagation constant
ρ	Voltage reflection coefficient
t	Time
U	Identity matrix
SE	Sending end of the transmission line
RE	Receiving end of the transmission li

Subscripts

S,R	Sending and receiving end quantifies
pf	Pre-fault value
m	Modal component

Superscripts

a,b,c,d,e,f	Phases
0,1,2,3,4,5	Modes
m	Modal component.

ABSTRACT

Higher phase order transmission (HPOT) is an important area of research in power systems. This is mainly due to the increasing demands for power which have forced engineering to explore newer avenues of power generation and transmission. Design of such multiphase systems requires that adequate protection schemes be also provided for. The thesis presents one such scheme for protection of multiphase systems. The scheme is known as ultra high speed protection relaying. It is based on the concept of travelling waves which are generated when a fault occurs on a system.

Rapid fault clearing has many advantages. Important among these is the improvement in transient stability of the system because the kinetic energy stored in the system varies as the square of the clearing time. In the past ultra high speed relaying was not feasible because of absence of fast acting circuit breakers. With advances in breaker technology, UHS relaying has become feasible.

The thesis discusses a frequency domain model for six phase transmission line. This model is suited to study of travelling wave generation and propagation. Fault analysis of multiphase systems has been studied and studies conducted on a sample power system. An

amplitude comparison relaying scheme based on travelling waves has been discussed along with its digital implementation.

CHAPTER 1

INTRODUCTION

High phase transmission (HPOT) system is receiving increasing attention today as a response to the ever growing demands for power. Increase of power transmission by increasing system voltage seems to have reached a saturation point. High voltage has many problems associated with it. These include, corona discharge, radio interference and also biological hazards. But the most important reason which prompted research into multiphase systems is the difficulty in obtaining right of way. Due to increasing pressure on land and its increasing costs, public utilities are finding it increasingly difficult to obtain new rights of way for transmission lines. It is hoped that development of multiphase systems would alleviate this problem to a large extent.

Adequate protection schemes are essential for proper operation of any power system. Any protective scheme must satisfy the dual requirements of speed and reliability. In this context, ultra high speed relays have gained increased importance. However, in the past use of such relays was hindered because of the absence of ultra high speed circuit breakers. But with recent developments [1,2], realisation of

UHS protective relaying schemes has become imperative.

Design of a relaying scheme requires that the system under study be accurately modelled. Ultra high speed relays discussed in the thesis use the concept of travelling waves generated during a fault. In other words, the relays are based on the transient waveforms generated after a fault. As a result, such relays have a distinct advantage over the conventional distance relays. Distance relays are based on calculation of impedance using the fundamental sinusoidal component of complex post fault waveform. This requires that, high frequency components and the dc offset be filtered out. This requires development of accurate analog/digital filters which results in time delays. On the other hand, travelling wave relays use the transient waveform as their input and hence, only the prefault fundamental needs to be filtered out. As a result, the relays are fast in operation.

The thesis aims at developing a new ultra high speed relaying scheme for the protection of multiphase systems based on the concept of travelling waves. For this the following approach has been adopted.

1. Mathematical modelling of a six phase transmission line for studying generation and propagation of travelling waves.

2. Multi-node fault analysis of power systems using the bus admittance matrix taking into account parameters like fault resistance and fault initiation angle.
3. Development of a relaying scheme for protection of a six phase transmission line using the travelling waves as relay input signals.
4. Digital implementation of the proposed relaying scheme.

Digital simulation of faulted transmission lines, (3-phase) has been discussed by Johns and Agarwal [3], Kothari et al. [4] and Humpage et al. [5]. Fault analysis techniques for three phase systems were discussed by Johns and Agarwal.

A novel protection scheme based on travelling wave concept was first discussed by Vittins [6]. Later work in this area was done by Chamia and Liberman [7], Matele [8] among others. Johns and Agarwal [9] also proposed a scheme which operates in conjunction with a carrier communication channel and in carrier blocking mode.

A differential relay based on the concept of travelling waves along with its theory, sensitivity analysis etc. was proposed by Takagi [10] and Akimoto et al. [11]. Digital protection schemes were studied

by Mann and Morrison [12,13]. Ramamoorthy and Verma [14] described a relaying principle in which a discriminant is used to determine the direction of the fault.

UHS relaying scheme developed by ASEA, Sweden have been listed at the Bonneville Power Administration (BPA) 500 kV system.

This thesis in all contains four chapters, starting with the present chapter One on Introduction.

The chapter Two develops a model for six phase transmission line suitable for transient analysis. Frequency domain Laplace transform has been used for this purpose. An equivalent Pi-model which is suited to multi-node analysis has been derived. The chapter also presents techniques for simulating various types of faults on a six phase system using the superposition principle. The frequency domain bus admittance matrix has been used to study the fault generated voltages and currents. A sample 3-bus six phase system has been used to test the techniques discussed in the chapter. The waveforms of fault generated voltages at relay location for various values of fault impedance has been presented. A new technique for Laplace transform inversion has also been presented.

The Chapter 3 presents the proposed ultra high speed relaying scheme. The scheme uses amplitude

comparison of the two relay signals to decide whether the fault is internal or external. The relay input signals are derived from the modal voltages and currents. The chapter also discusses the digital implementation of the proposed relaying scheme. Finally the relaying scheme is simulated on a sample 3-bus power system.

Finally Chapter 4 concludes with a review of the work presented in the thesis and scope for future work.

CHAPTER 2

DYNAMIC MODELLING AND TRANSIENT FAULT ANALYSIS OF MULTIPHASE SYSTEMS

2.1 INTRODUCTION

The chapter presents a model for a six phase transmission line which is suitable for transient analysis and travelling wave study. The model is developed in the Laplace S-domain. An algorithm which is used for inversion to time domain is also presented. The chapter also presents fault simulation of various types of faults on a six phase system and the algorithm for fault analysis of multinode systems.

The model for a six phase transmission line developed is the S-domain π-model which is suitable for nodal analysis. In the past, frequency domain studies have been largely confined to the use of Fourier transforms. This was because Laplace transform inversion was cumbersome and the computations were unwieldy. The chapter discusses a new scheme for inversion of the Laplace transform. The algorithm discussed reduces Laplace inversion to a matrix multiplication. The fault simulation invokes the superposition principle to calculate the fault generated components of voltages and currents.

To design a protection scheme for a given power system requires accurate determination of post fault voltages and currents. For this, accurate modelling of various power system elements is needed along with computational techniques which are numerically accurate as well as efficient. To this end, frequency domain analysis has been found to be more accurate [15].

Modelling and fault simulation of three phase systems has been carried out by Humpage et al. [5], Johns and Agarwal. However, these models suffer from the drawback that they are cumbersome and can be applied with ease only to simple systems. Modelling of multiphase power system elements has been done by Tiwari and Singh [16] and Chowdhary and Singh [17]. Most of these models are meant for steady state analysis such as load flow and planning studies etc.

2.2 FREQUENCY DOMAIN PI-MODEL OF SIX PHASE TRANSMISSION LINE

For simplicity in analysis, we assume that the line is perfectly transposed. This would imply,

$$Z_{aa} = Z_{bb} = Z_{cc} = Z_{dd} = Z_{ee} = Z_{ff}$$

$$Z_{ab} = Z_{bc} = Z_{cd} = Z_{de} = \dots = Z_{fa}.$$

Let the column vector [V] denote the vector of the six phase to ground voltages, i.e.

$$[V] = \begin{bmatrix} V_a(s) \\ V_b(s) \\ V_c(s) \\ V_d(s) \\ V_e(s) \\ V_f(s) \end{bmatrix}$$

Here, s is the Laplace domain parameter. Similarly, let [I] denote the column vector of six line currents in the six phases

$$[I] = \begin{bmatrix} I_a(s) \\ I_b(s) \\ I_c(s) \\ I_d(s) \\ I_e(s) \\ I_f(s) \end{bmatrix}$$

Then, in the Laplace domain, the transmission line equations can be written as,

$$d[V]/dx = -[Z][I] \tag{1}$$

$$d[I]/dx = -[Y][V] \tag{2}$$

x is the transmission line's length parameter.
Here [Z] is the impedance (per unit length) matrix

admittance (p.u. length) matrix, i.e.

$$[Z] = \begin{bmatrix} Z_{aa} & Z_{ab} & \cdots\cdots & Z_{af} \\ Z_{ba} & Z_{bb} & & \cdot \\ \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot \\ Z_{fa} & \cdots\cdots & & Z_{ff} \end{bmatrix}$$

and, similarly

$$[Y] = \begin{bmatrix} Y_{aa} & Y_{ab} & \cdots\cdots & Y_{af} \\ Y_{ba} & Y_{bb} & & \cdot \\ \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot \\ Y_{fa} & \cdots\cdots & & Y_{ff} \end{bmatrix}$$

As stated above for a perfectly transposed line, we have,

$$Z_{aa} = Z_{bb} = Z_{cc} = \cdots\cdots = Z_{ff} = Z_s$$

$$Z_{ab} = Z_{bc} = Z_{cd} = \cdots\cdots = Z_{fa} = Z_m.$$

Using equations (1) and (2) above, we get

$$d^2[V]/dx^2 = [Z][Y][I] \qquad (3)$$

Equation (3) consists of six coupled ordinary differential equations. We should decouple these equations so that closed from solution can be obtained.

It is easily seen, that, for a transposed line (assuming perfectly transposed line), the matrix,

$$[P] = [Z]\,[Y]$$

is symmetric matrix whose off diagonal elements are equal and the diagonal elements are also equal. This suggests, that, we can use the linear transformation matrix (similar to symmetrical component transformation matrix for a 3-phase line)

$$[T] = \frac{1}{\sqrt{6}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & a & a^2 & -1 & -a & -a^2 \\ 1 & a^2 & -a & 1 & a^2 & -a \\ 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -a & a^2 & 1 & -a & a^2 \\ 1 & -a^2 & -a & -1 & a^2 & a \end{bmatrix} \qquad \begin{aligned} a &= \exp(-j60^{o}) \\ &= 6\sqrt{1} \end{aligned} \tag{4}$$

to decouple the above equations (3). This transformation matrix [T] is linear, power invarient and complex. These decoupled equations are of the form,

$$\frac{d^2V^{(m)}}{dx^2} = \sigma_m V^{(m)} \qquad m = 0,1,2,3,4,5 \qquad (5)$$

$V^{(m)}$ is called the m^{th} modal voltage and σ_m is the m^{th} mode propagation constant. Actually, σ_m represents the m^{th} eigenvalue of the [P] matrix. Each of the equations (5) can be solved independently. Correspondingly, the modal current equations are,

$$\frac{d^2I^{(m)}}{dx^2} = \sigma_m I^{(m)} \qquad m = 0,1,2,3,4,5 \qquad (6)$$

solving equation (5) we get,

$$V^{(m)}(s) = K_1 \cosh \sigma_m x + K_2 \sinh \sigma_m x$$
$$m = 0,1,2,3,4,5 \qquad (7)$$

Then, using equation (1) we get,

$$I^{(m)}(s) = \frac{-\sigma_m}{Z^{(m)}} (K_1 \sinh \sigma_m x + K_2 \cosh \sigma_m x)$$
$$m = 0,1,2,3,4,5 \qquad (8)$$

Here, $Z^{(m)}$ is the m^{th} eigenvalue of the [Z] matrix. The boundary conditions at the receiving end are,

$$I^{(m)} = I_R^{(m)} \text{, and } V^{(m)} = V_R^{(m)} \text{ at } x = l.$$

Similarly, at the sending end,

$$I^{(m)} = I_S^{(m)} \text{ and } V^{(m)} = V_S^{(m)} \text{ at } x = 0.$$

Using these boundary conditions, we get, for each modal component,

$$\begin{bmatrix} V_R^{(m)}(s) \\ I_R^{(m)}(s) \end{bmatrix} = \begin{bmatrix} \cosh \sigma_m \ell & \dfrac{-Z^{(m)}}{\sigma_m} \sinh \sigma_m \ell \\ \dfrac{-\sigma_m}{Z^{(m)}} \sinh \sigma_m \ell & \cosh \sigma_m \ell \end{bmatrix} \begin{bmatrix} V_S^{(m)}(s) \\ I_S^{(m)}(s) \end{bmatrix} \tag{9}$$

Finally, the modal values are related to the corresponding phase values by the relation

$$\begin{aligned} [V] &= [T]\,[V]_m \\ [I] &= [T]\,[I]_m \end{aligned} \tag{10}$$

$[V]_m$ and $[I]_m$ are the modal voltage and modal current column vectors respectively.

Using the above transformation, we get, the following equations

$$\begin{aligned} [V]_R &= [T][\cosh \sigma\ell][T]^*[V]_S - [T][Z_M][\sigma]^{-1}[\sinh \sigma\ell][T]^*[I_S] \\ [I]_R &= -[T][\sigma][Z_M]^{-1}\,[\sinh \sigma\ell][V_S] - [T][\cosh \sigma\ell][T]^*[I_S] \end{aligned} \tag{11}$$

where

$[V]_R$ is the receiving end phase voltage vector;

$[V]_S$ is the sending end phase voltage vector;

$[I_R]$ is the receiving end line current vector;

$[I_S]$ is the sending end line current vector;

$[\sigma]$ diag$[\sigma_o\ \sigma_1\ \sigma_2\ \sigma_3\ \sigma_4\ \sigma_5]$

$[\cosh \sigma\ell] = \mathrm{diag}[\cosh\sigma_o\ell\ \ \cosh \sigma_1\ell\ \ \cosh \sigma_2\ell\ \ \cosh \sigma_3\ell$

$\cosh \sigma_4\ell\ \ \cosh \sigma_5\ell]$

$$[\sinh \sigma\ell] = \text{diag}[\sinh \sigma_o\ell \ \sinh \sigma_1\ell \ \sinh \sigma_2\ell \ \sinh \sigma_3\ell \ \sinh \sigma_4\ell \ \sinh \sigma_5\ell\,]$$

and,

$$[Z_M] = \text{diag}\,[Z^{(o)} \ Z^{(1)} \ Z^{(2)} \ Z^{(3)} \ Z^{(4)} \ Z^{(5)}]$$

The above equations can be written in compact form as shown,

$$\begin{bmatrix} V_R^{a,b,c,d,e,f} \\ I_R^{a,b,c,d,e,f} \end{bmatrix} = \begin{bmatrix} [A'] & [B'] \\ [C'] & [D'] \end{bmatrix} \begin{bmatrix} V_S^{a,b,c,d,e,f} \\ I_S^{a,b,c,d,e,f} \end{bmatrix} \quad (12)$$

The inverse equation can be written as,

$$\begin{bmatrix} V_S^{a,b,c,d,e,f} \\ I_S^{a,b,c,d,e,f} \end{bmatrix} = \begin{bmatrix} [A] & [B] \\ [C] & [D] \end{bmatrix} \begin{bmatrix} V_R^{a,b,c,d,e,f} \\ I_R^{a,b,c,d,e,f} \end{bmatrix} \quad (13)$$

where

$$\begin{aligned} [A] &= [T]^* \, [\cosh \sigma\ell] \, [T] = [D] \\ [B] &= [T]^* \, [Z_M] \, [\sigma]^{-1} \, [\sinh \sigma\ell] \, [T] \\ [C] &= [T]^* \, [\sigma] \, [\sinh \sigma\ell] \, [Z_M]^{-1} \, [T] \end{aligned} \quad (14)$$

The equation (13) represents the frequency S-domain equations which relate the receiving and sending end voltages and currents in Laplace domain.

Equation (13) forms the basis of the S-domain Pi-model of the six phase transmission line.

Consider the Pi-model of the six phase transmission line as shown in the Fig. 1. From the model, we can write the following equations relating receiving and sending end quantities

$$V_S^{a,b,c,d,e,f} = (U + Z_P^{a,b,c,d,e,f}\, Y_{2P}^{a,b,c,d,e,f})V_R^{a,b,c,d,e,f} + Z_P^{a,b,c,d,e,f}\, I_R^{a,b,c,d,e,f}$$

$$I_S^{a,b,c,d,e,f} = (Y_{2P}^{a,b,c,d,e,f} + Y_{1P}^{a,b,c,d,e,f} + Y_{1P}^{a,b,c,d,e,f}\, Z_P^{a,b,c,d,e,f}\, Y_{2P}^{a,b,c,d,e,f})\, V_R^{a,b,c,d,e,f} + (U + Y_{1P}^{a,b,c,d,e,f}\, Z_P^{a,b,c,d,e,f})I_R^{a,b,c,d,e,f} \qquad (15)$$

where,

$Z_P^{a,b,c,d,e,f}$ = six phase transform impedance matrix of the series branch

$Y_{1P}^{a,b,c,d,e,f}$, $Y_{2P}^{a,b,c,d,e,f}$ = six phase transform admittance matrix of the shunt branch

U = Identity i.e. unit matrix of the order 6.

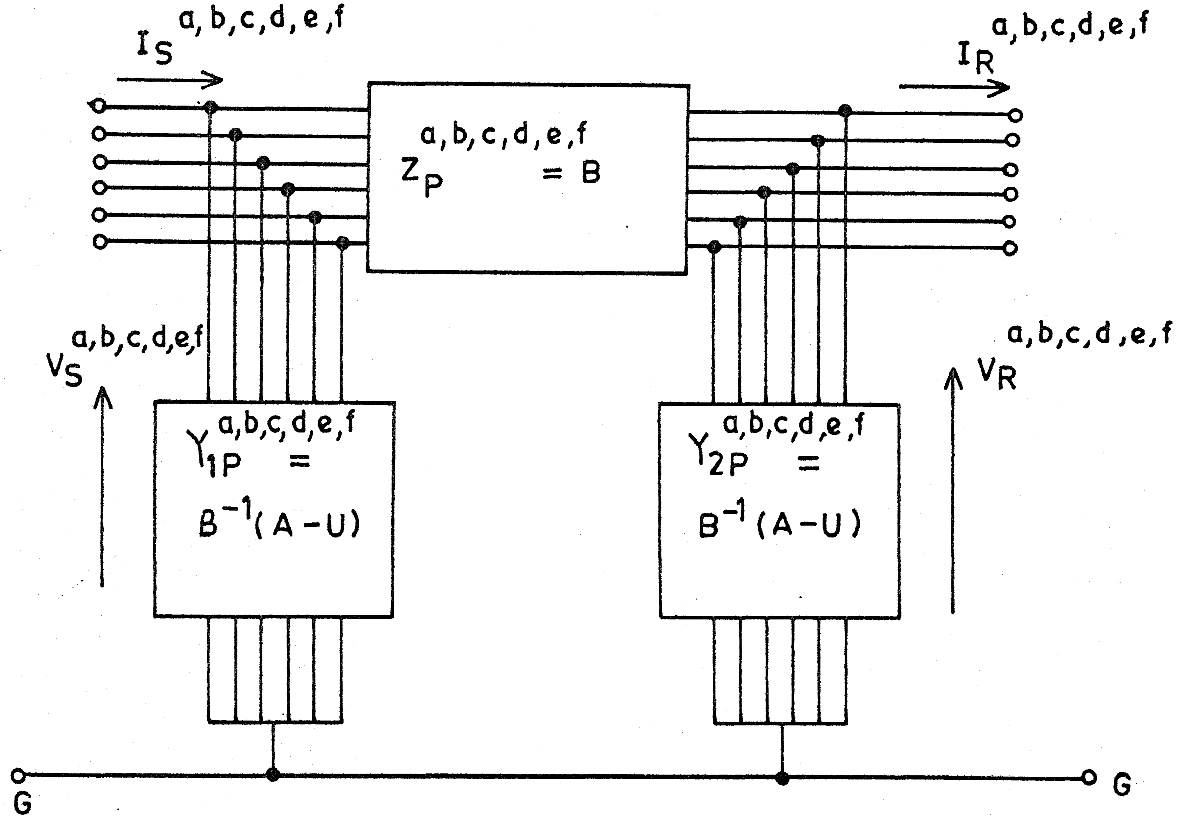


FIG. 1 FREQUENCY DOMAIN PI MODEL OF SIX PHASE TRANSMISSION LINE

Comparing equations (13) and (15) and, we get

$$Z_P^{a,b,c,d,e,f} = [B]$$

$$Y_{1P}^{a,b,c} = ([D] - U)\,[B]^{-1}$$

$$Y_{2P}^{a,b,c} = [B]^{-1}\,([A] - U)$$

By expanding the matrices, it is easily seen, that, $Y_{1P} = Y_{2P}$. Thus we finally have the parameters of the frequency domain Pi-model of a six phase transmission line

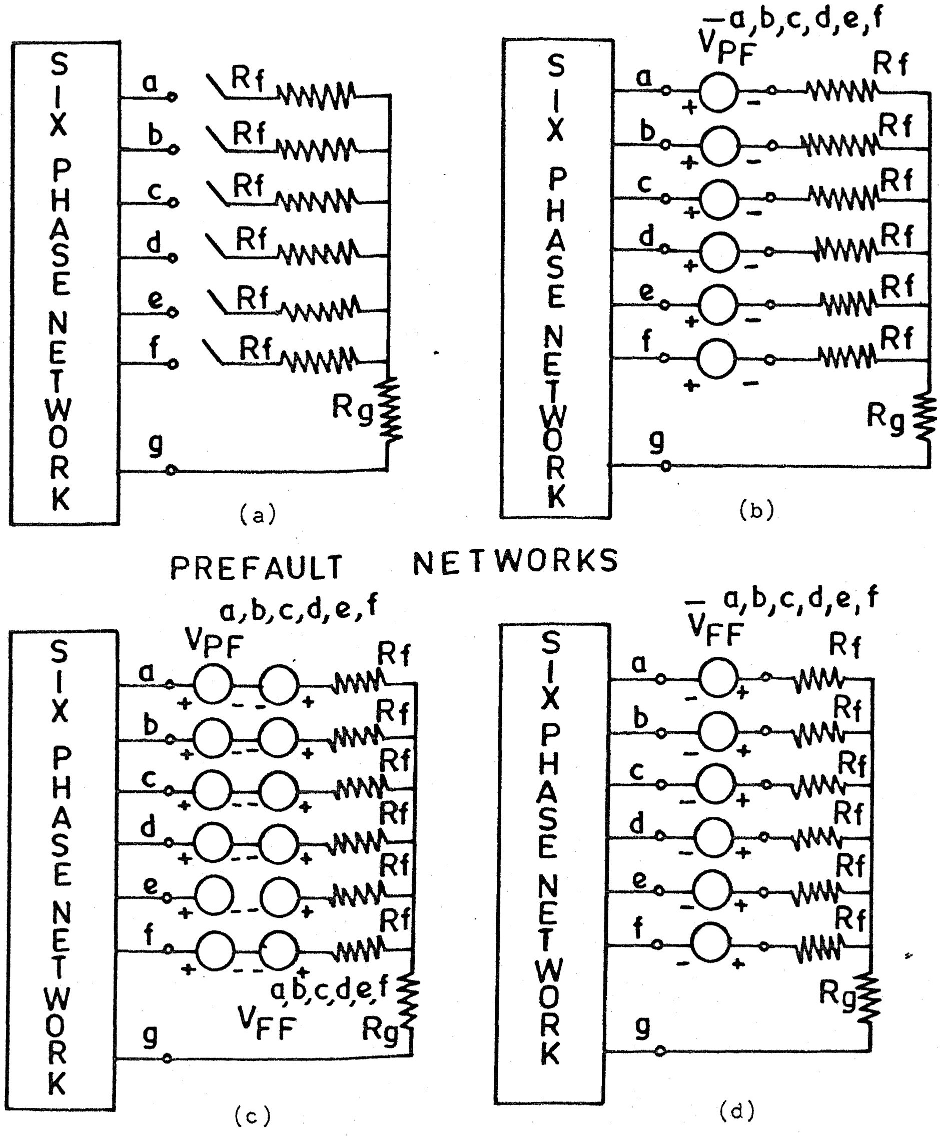
$$Z_P^{a,b,c,d,e,f} = [B]$$

$$Y_{1P}^{a,b,c,d,e,f} = Y_{2P}^{a,b,c,d,e,f} = [B]^{-1}\,([A]-U) \tag{16}$$

2.3 FAULT SIMULATION OF A SIX PHASE POWER SYSTEM

We are interested in calculating the fault generated components of voltages and currents. For this, the principle of superposition is used. The prefault values of voltages and currents can be found through load flow studies. Obviously, the fault generated voltage and current waves shall depend on the prefault values and the fault initiation angle.

Consider the six phase network shown in Fig.2(a). Only the faulted node is shown in detail. The open six pole switch represents the unfaulted condition.



POST FAULT NETWORK

NETWORK TO EVALUATE FAULT GENERATED COMPONENTS

FIG. 2

The network remains unfaulted by connecting a six phase voltage source $[V_{PF}]$ as shown. $[V_{PF}]$ represents the prefault voltage vector. Now the fault is simulated by connecting a voltage source $[V_{FF}]$ with the polarity as shown in the figure. The value of $[V_{FF}]$ depends on the type of fault, e.g. for a six phase fault $[V_{FF}] = [V_{PF}]$. For other types of faults too $[V_{FF}]$ can be calculated from the fault conditions.

To calculate only the fault generated components, we short the voltage source $[V_{PF}]$ as also all the independent voltage sources in the system. This is shown in Fig. 2(d). At the faulted bus, there is a series branch of the voltage source $[V_{FF}]$ and the series fault resistance $[R_F]$. It is easily seen, that, the element of $[R_F]$ are,

$$
\begin{aligned}
[R_F]_{ij} &= R_f + R_g & i = j \\
&= R_g & i \neq j
\end{aligned}
$$

For nodal analysis, it is more convenient to convert the voltage source $[V_{PF}]$ and impedance $[R_F]$ to an equivalent current norton source. This is shown in Fig. 3.

$$
\begin{aligned}
[J_F] &= [G_F]\,[V_{FF}] \\
[G_F] &= [R_F]^{-1} \qquad (17)
\end{aligned}
$$

Now the augmented bus admittance matrix for the power system can be derived. With $[J_F]$ as the only

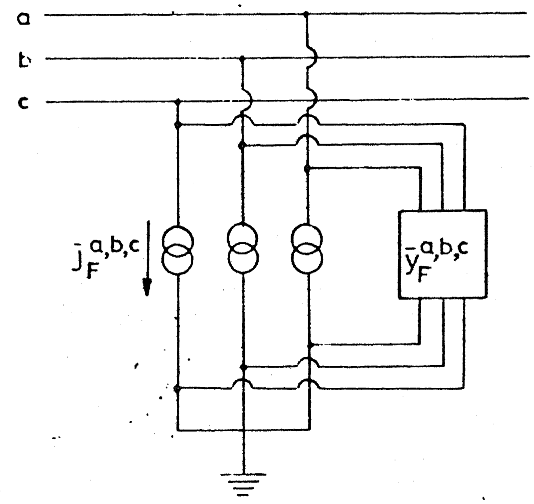
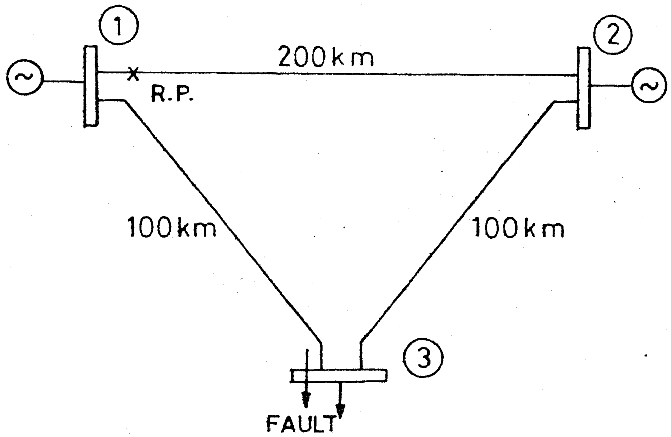


Fig. 3 EQUIVALENT CURRENT SOURCE REPRESENTATION AT THE FAULT BUS



R.P. - Relay point

FIG. 4 A SAMPLE 3-BUS POWER SYSTEM

independent current source in the system, the fault generated components of voltages and currents can be found. It should be noted, that, all these values are in the transform domain and have yet to be converted to time domain values.

2.4 INVERSION TO TIME DOMAIN

Use of the Laplace domain in the past has been restricted because of problems in inverting it. Techniques used in the past [18] have had the drawback of excessive storage requirements and required time consuming algorithms.

A new algorithm for inverting the Laplace transform has been proposed based on quadrature [19].

We know that the Laplace transform of a time function u(t) is given by,

$$F(s) = \int_{0}^{\infty} u(t)\, e^{-st}\, dt \qquad (18)$$

Further, to an approximation, closed form integration can be replaced by a summation, i.e., an integral of the type

$$\int_{-1}^{1} f(r)\, dr = \sum_{i=1}^{N} W_i\, f(r_i) \qquad (19)$$

It can be shown that any function f(r) in the range (-1, 1) can be expressed in terms of a set of Legendre Polynomials in this region, i.e.

$$f(r) \cong \sum_{k=0}^{N} b_k P_k(r) \qquad r = [-1,1] \qquad (20)$$

This follows from the following two properties of Legendre polynomials, in the region (-1,1).

(i) Orthogonality, and

(ii) Completeness.

However, by choosing ω_i and r_i suitably, we can make eqn. (19) to be exact for any polynomial of degree less than or equal to 2N-1.

We shall show, that,

the r_i, $i = 1,2....N$ are the zeros of $P_N(r)$

where $P_N(r)$ is Legendre polynomial of degree N. (21)

$$\omega_i = \int_{-1}^{1} \frac{P_N(r)\,dr}{(r-r_i)P_N'(r_i)} \qquad i = 1,2,.....N \quad (22)$$

To prove the first statement, consider the polynomials $r^k P_N(r)$ $k = 0,1,2,....,N-1$. Then,

$$0 = \int_{-1}^{1} r^k P_N(r)\,dr = \sum_{i=1}^{N} \omega_i r_i^k P_N(r_i) \quad k = 0,1,2,...N-1.$$

Consider these as N linear equations in $\omega_i P_N(r_i)$. Then the determinant,

$$|r_i^k| \qquad i = 1,2....N \qquad k = 0,1,....N-1$$

the Vandermonde matrix is non zero as all r_i's are distinct. Hence, the only solution is the trivial one, i.e.

$$\omega_i P_N(r_i) = 0 \quad i = 1,.....N.$$

Since, $\omega_i \neq 0$, we have,

$$P_N(r_i) = 0 \quad i = 1,.....N.$$

Hence, r_i's are the zeros of $P_N(r)$.

The second equation can be verified by substituting it in eqn. (19).

Finally, we make the change of variables, i.e.

$$\frac{1+r}{2} = x \qquad r = 2x - 1$$

then, the quadrature formula becomes,

$$\int_0^1 g(x)dx = \sum_{i=1}^{N} \omega_i \, g(x_i) \qquad g(x) = f(2x-1)$$

x_i's are zeros of the shifted Legendre polynomial $P_N(2x - 1)$. We are interested in inverting eqn. (18). For this, we first map the $[0, \infty]$ region to $[0, 1]$ region by using the change of variable $x = e^{-t}$. Then, we have,

$$F(s) = \int_0^1 x^{s-1} \, g(x) \, dx \qquad g(x) = u(-\log x)$$

or $$\sum_{i=1}^{N} \omega_i x_i^k \, g(x_i) = F(k+1) \quad k = 0,1,2....N+1 \qquad (23)$$

Now ω_i's and x_i^k are known before hand for a given N. Hence, we can calculate these and denote them by matrix M.

where,

$$M = [\omega_i x_i^k]_{ik}$$

We want to find $g(x_i)$ for $i = 1,2....N$. $F(k+1)$ are all known for $k = 0,1,2....N-1$ as $F(s)$ is known. Hence,

$$[g(x_i)] = M^{-1} [F(k+1)] \tag{24}$$

M^{-1} can be calculated and stored for given N. Then, all that has to be done to find the inverse Laplace transform of F(s) is to multiply this inverse matrix by [F(k+1)] column vector. The inverse matrix is given in the Appendix along with the time instants $t_i = - \log x_i$, $i = 1,.....N$. To find the time function at other time instants, use the scaling factor, i.e.,

$$(f(at)) = \frac{F(s/a)}{a}$$

2.5 NUMERICAL EXAMPLE

A sample 3-bus power system as shown in Fig. 4 is considered to study the fault generated travelling waves on a six phase transmission line. The fault is assumed to have occurred on the middle of one of the transmission lines. Identical sources are considered at either end of the line.

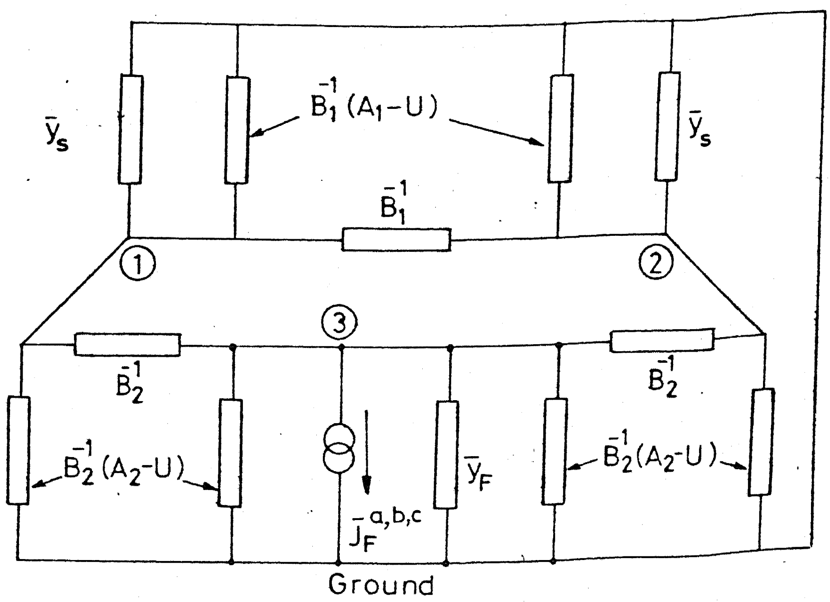


FIG. 5 ONE-LINE DIAGRAM OF THE EQUIVALENT NETWORK OF 3-BUS POWER SYSTEM

The one line diagram of the equivalent network of the sample power system is shown in Fig. 5. $[A_1]$, $[B_1]$, $[C_1]$ and $[D_1]$ are the line parameters for length 200 km and $[A_2]$, $[B_2]$, $[C_2]$ and $[D_2]$ are the corresponding matrices for length 100 km. By inspection, the bus admittance matrix is given by

$$Y_{BUS}^{a,b,c,d,e,f} = \begin{bmatrix} B_1^{-1}A_1+B_2^{-1}A_2+[\overline{Y}_S] & -B_1^{-1} & -B_2^{-1} \\ -B_1^{-1} & B_1^{-1}A_1+B_2^{-1}A_2+[Y_S] & -B_2^{-1} \\ -B_2^{-1} & -B_2^{-1} & 2B_2^{-1}A_2+[Y_F] \end{bmatrix}$$

We are interested in finding the voltage and current values at the relay locations. These are to be found in terms of $[V_{FF}]$.Solving the nodal equations, we get,

$$[V_1] = [2B_2^{-1}-(2B_2^{-1}A_2+[Y_F])B_2(B_2^{-1}A_1+B_2^{-1}A_2+[Y_S]-B_1^{-1})^{-1}[Y_F][V_{FF}]$$

and

$$[I_{12}] = B_1^{-1}(A_1-U)[V_1].$$

Since, the travelling waves are generated only during the most severe faults, only the six phase to ground is considered. For this fault, $[V_{FF}] = [V_{PF}]$ i.e.

$$[V_{FF}] = \frac{V_m}{S^2+\omega_o^2} \begin{bmatrix} s\ \sin \phi_o + \omega_o \cos \phi_o \\ s\ \sin(\phi_o - 60^o) + \omega_o \cos(\phi_o - 60^o) \\ s\ \sin(\phi_o - 120^o) + \omega_o \cos(\phi_o - 120^o) \\ -s\ \sin(\phi_o) - \omega_o \cos \phi_o \\ s\ \sin(\phi_o + 120^o) + \omega_o \cos(\phi_o + 120^o) \\ s\ \sin(\phi_o + 60^o) + \omega_o \cos(\phi_o + 60^o) \end{bmatrix}$$

ω_o = natural system frequency.

V_m = maximum value of prefault voltage = 1.0 pu.

ϕ_o = fault initiation angle.

Using the above $[V_{FF}]$ vector, the voltage and current values are found at relay location 1. Then, using the inversion matrix, corresponding time domain values are found.

The waveforms for various values of ϕ_o and fault impedance are shown in the Figs.6,7 and 8.

2.6 RESULTS AND DISCUSSIONS

The chapter presents a frequency domain model suitable for study of transient phenomenon on six phase transmission line. The equivalent Pi-model is suitable for the nodal analysis of six phase power systems using the bus admittance matrix. The approach taken is

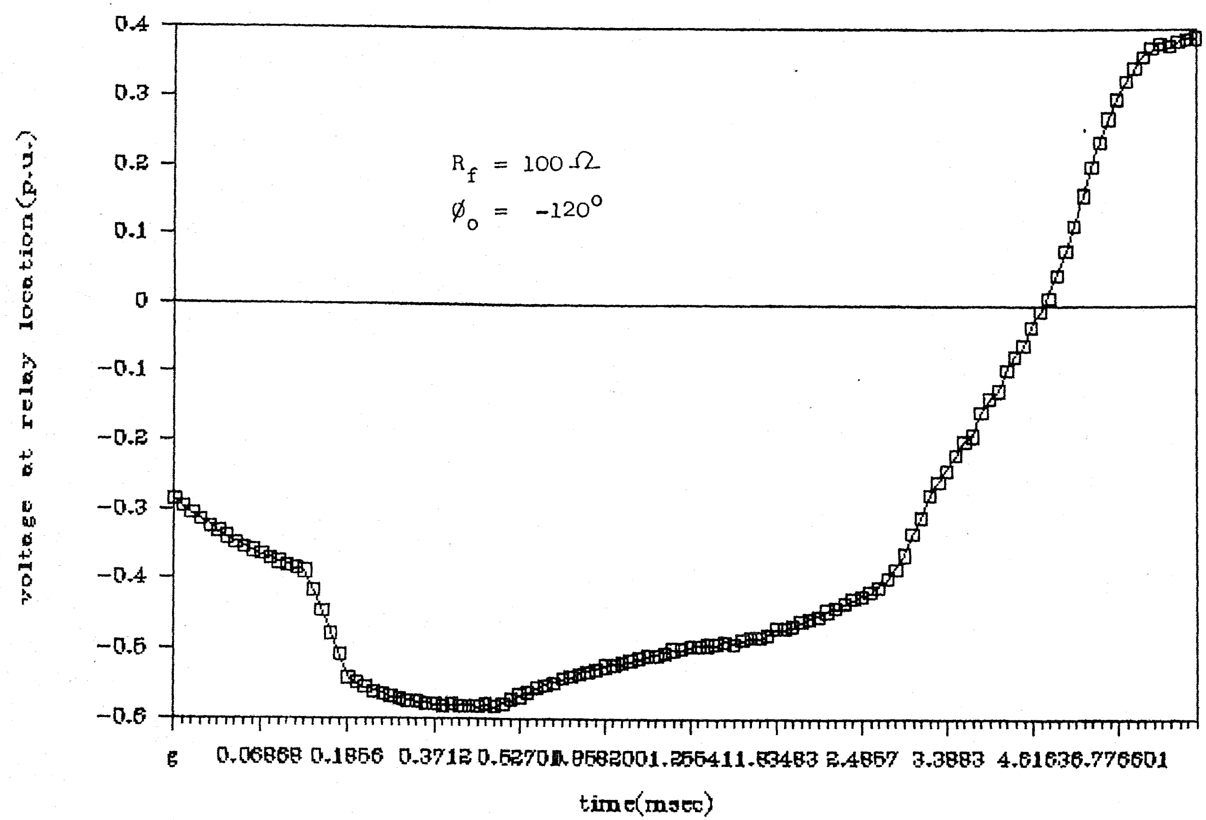


Fig. 6 Fault generated voltage waveforms at relay location (ϕ_o =-120°)

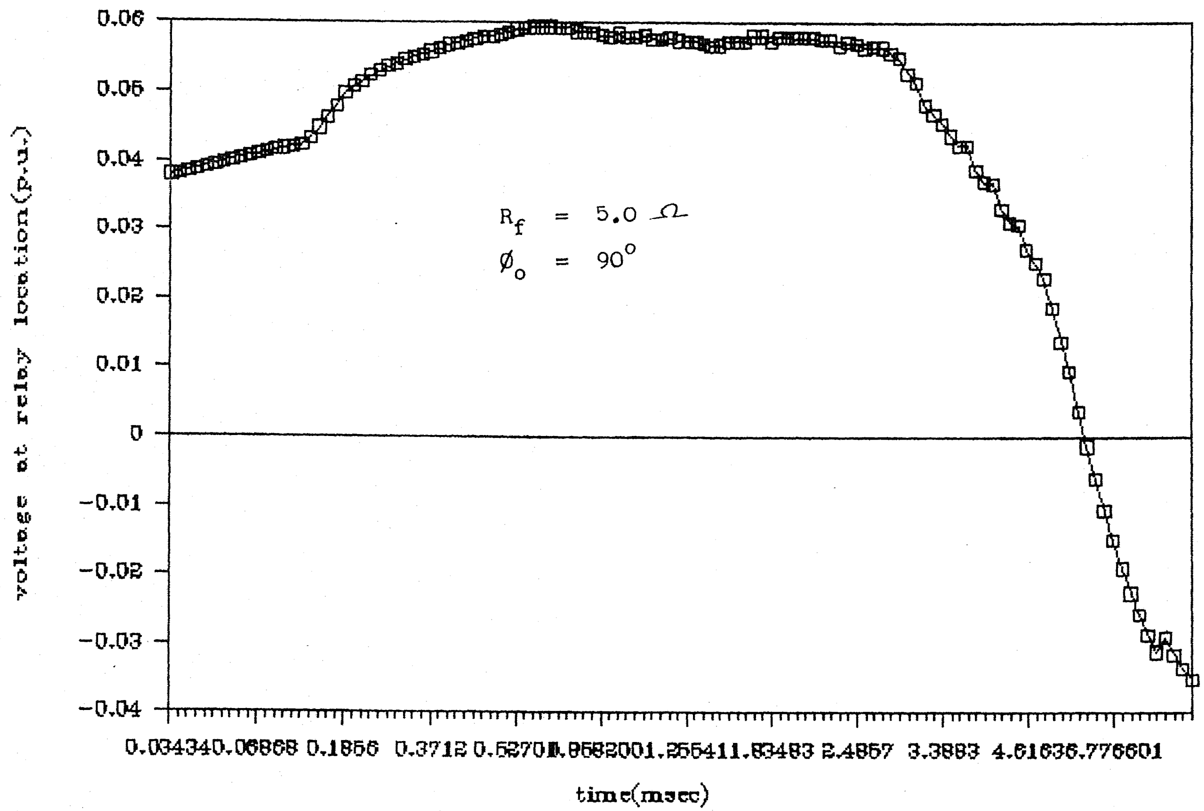


Fig. 7 Fault generated voltage waveform at relay location ($\phi_o = 90^\circ$)

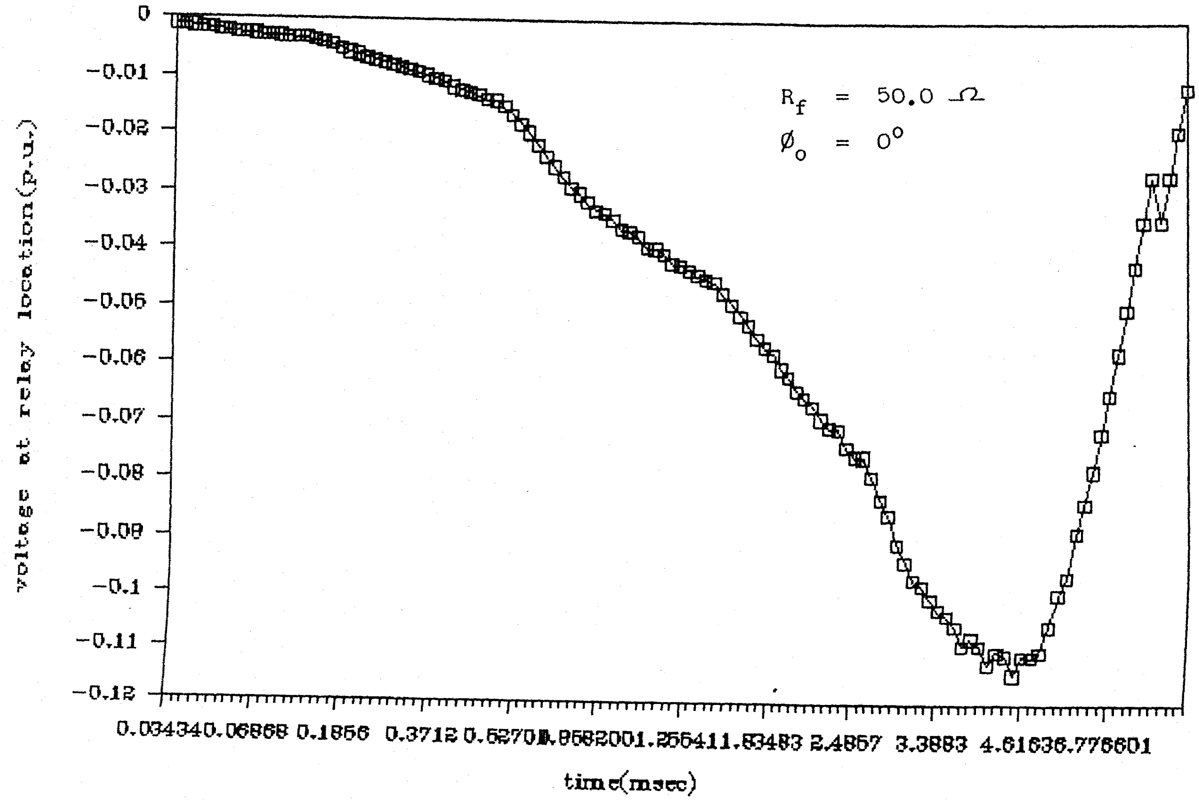


Fig. 8 Fault generated voltage waveform at relay location ($\phi_o = 0^{\circ}$)

completely general and can be used for study of faults on large systems too.

The chapter presents a novel technique for inversion of the Laplace transform. The technique based on quadrature principle shows how the inversion process can be approximated as a matrix multiplication. The number of operations required to find the time function at n time instants varies as n^2. It is clear, that, the technique is superior in many respects to the conventional Fourier transform techniques. Further, no complex numbers are involved thus saving computational time further. Yet another advantage of the scheme is, that, it does not require much storage space. For example, the inverse matrix requires only the n^2 matrix elements to be stored along with the n zeros of the shifted Legendre polynomials. It is hoped, that, this technique would be found useful in transient analysis of power systems.

CHAPTER 3

PROPOSED UHS RELAYING SCHEME FOR SIX PHASE TRANSMISSION LINE BASED UPON TRAVELLING WAVE PHENOMENA

3.1 INTRODUCTION

Designing six phase systems would require alongside, adequate protection schemes which are fast in operation and reliable. Over the years, various protection schemes have been proposed based on steady state analysis as well as transient behaviour. In the recent years, travelling wave relaying schemes have been proposed. These relaying schemes come in the category of ultra high speed schemes, i.e. (it operates in less than quarter cycle). This ultra high speed clearing of fault would have a two fold advantage :- i) it would improve the transient stability of the system as the kinetic energy introduced into the system is proportion to the square of the fault clearing time [20]. Secondly, this also improves system reliability. In the past, the feasibility of ultra high speed relaying scheme was not very bright because of the absence of high speed circuit breakers. However, with recent advances in circuit breaker technology [1,2] attention is now focussed on ultra high speed protective relays.

These ultra high speed relays are based on the travelling wave phenomena which are generated on the occurrence of a fault and travel towards relays at either end from fault location.

Takagi et al. [10] and Akimoto et al. [11], defined a relaying signal, using sending and receiving end voltages and currents and also the surge impedance of the line. The main demerit is that the scheme requires exchange of information between ends of the protected line. Fault location scheme proposed by above authors are to complicated for easy realisation.

An ultra high speed relaying scheme developed by ASEA, Sweden has been tested on Bonville Power Administration (U.S.A.) (BPA) 500 kV system. It needs the exchange of only qualitative information between the ends of the protected line. Basically, the scheme is a directional comparison scheme. Fault location schemes have been proposed by Vittins [6]. Fault direction determination algorithms have also been proposed by Johns [9] and Agarwal.

In this chapter, an amplitude comparison relaying scheme based on travelling wave phenomenon has been proposed. The scheme utilises the reflection properties of travelling waves to derive appropriate

relaying signals [21]. The scheme proposed is simple and reliable. A major advantage is, that, it does not require any transfer of information between the ends of the protected line. The results of digital simulation of a faulted sample power system are presented to confirm the feasibility of the proposed scheme.

3.2 PRINCIPLE OF OPERATION

The principle of operation can be best studied by considering an equivalent single phase system. Consider the single phase transmission line connecting two large power systems as shown in Fig. 9. After the occurrence of the fault, the voltage and current at any point in the system can be regarded as the sum of prefault and fault generated values, i.e.

$$v = v_{pf} + v_f$$

$$i = i_{pf} + i_f$$

The fault generated components, travel from the fault location towards either end of the protected line. Assuming a loss less line, the fault generated components can be represented as

$$v_f = f_1(x-ut) + f_2(x+ut)$$

$$i_f = \frac{1}{Z_o}\left(f_1(x-ut) - f_2(x-ut)\right)$$

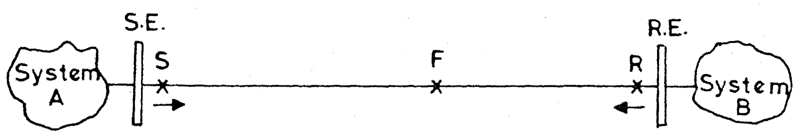


FIG. 9 A TRANSMISSION LINE INTERCONNECTING TWO POWER SYSTEMS

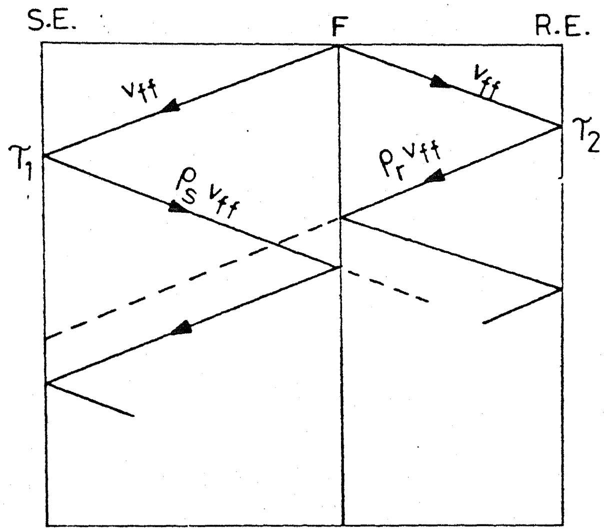


FIG.10 LATTICE DIAGRAM FOR AN INTERNAL FAULT

where,

u = velocity of propagation on the line

Z_o = surge impedance of the line.

These travelling waves on reaching the relay locations undergo reflection and refraction. If ρ_s and ρ_r be the reflection coefficients at either end of the line, the subsequent waves on the line can be represented by a Bewley diagram shown in Fig. 10. v_{ff} is the fault generated component of the voltage at the fault point. For a bolted fault it is the prefault voltage at the point of fault.

The proposed relaying scheme uses the following pair of signals at each relay location. For the sending end relay, the input signals in p.u. would be

$$S_1 = v_{fs} - R_{set}\, i_{fs}$$

$$S_2 = v_{fs} + R_{set}\, i_{fs}$$

v_{fs} and i_{fs} are the fault generated components of voltage and current at sending end, and,

R_{set} = surge impedance of the line.

For the faulted line, the values of v_{fs}, i_{fs}, $|S_1|$ and $|S_2|$ at various instants have been shown in Table 1. τ represents the time of travel of the wave from the fault location to the relay location. Since magnitude of ρ_s

is less than 1.0, it is seen that $|S_1| > |S_2|$ from $t = \tau_1$ till a further reflected wave appears at the relay location.

Now, consider a fault which occurs outside the protected zone. When the travelling wave schemes the relay location, it gets reflected and refracted at the sending end bus. Due to the refracted wave we have,

$$v_{fs} = \rho_t \, v_{ff} \qquad i_{fs} = \rho_t \, v_{ff}/Z_o' \tag{26}$$

where

ρ_t = refraction coefficient at the sending end bus

Z_o' = surge impedance of the faulted line.

Then, we have,

$$S_1 = \left(1 - \frac{R_{set}}{Z_o'}\right) \rho_t \, v_{ff}$$

$$S_2 = \left(1 + \frac{R_{set}}{Z_o'}\right) \rho_t \, v_{ff} \tag{27}$$

Hence, for some duration, in this case, $|S_1| < |S_2|$.
Thus we have the following result,

$$|S_1| > |S_2| \qquad \text{for internal fault}$$

$$|S_1| < |S_2| \qquad \text{for external fault.} \tag{28}$$

This property is used for designing an appropriate relaying scheme.

3.3 EXTENSION TO A SIX PHASE SYSTEM

From equations (5,6) we know that the transmission line equation of the six phase system can be written in decoupled form using the symmetrical transformation matrix [T], i.e.

$$\frac{d^2V^{(m)}}{dX^2} = \sigma_m V^{(m)} \qquad m = 0,1,2,3,4,5$$

$$\frac{d^2I^{(m)}}{dX^2} = \sigma_m I^{(m)} \qquad (29)$$

This suggests, that, the modes propagate independently of each other and that a six phase transmission line behaves as a single phase line for each of these modes separately. It follows, therefore, that the results obtained in the previous section, apply equally well to each modal component separately. Hence, six pairs of signals, one for each mode, are employed. For the sending end relay, these are given by,

$$S_1^{(m)} = v_{fs}^{(m)} - R_{set}^{(m)} \; i_{fs}^{(m)}$$

$$m = 0,1,2,3,4,5$$

$$S_2^{(m)} = v_{fs}^{(m)} + R_{set}^{(m)} \; i_{fs}^{(m)} \qquad (30)$$

where, $v_{fs}^{(m)}$, $i_{fs}^{(m)}$ are the modal component values of fault generated voltages and currents. These are

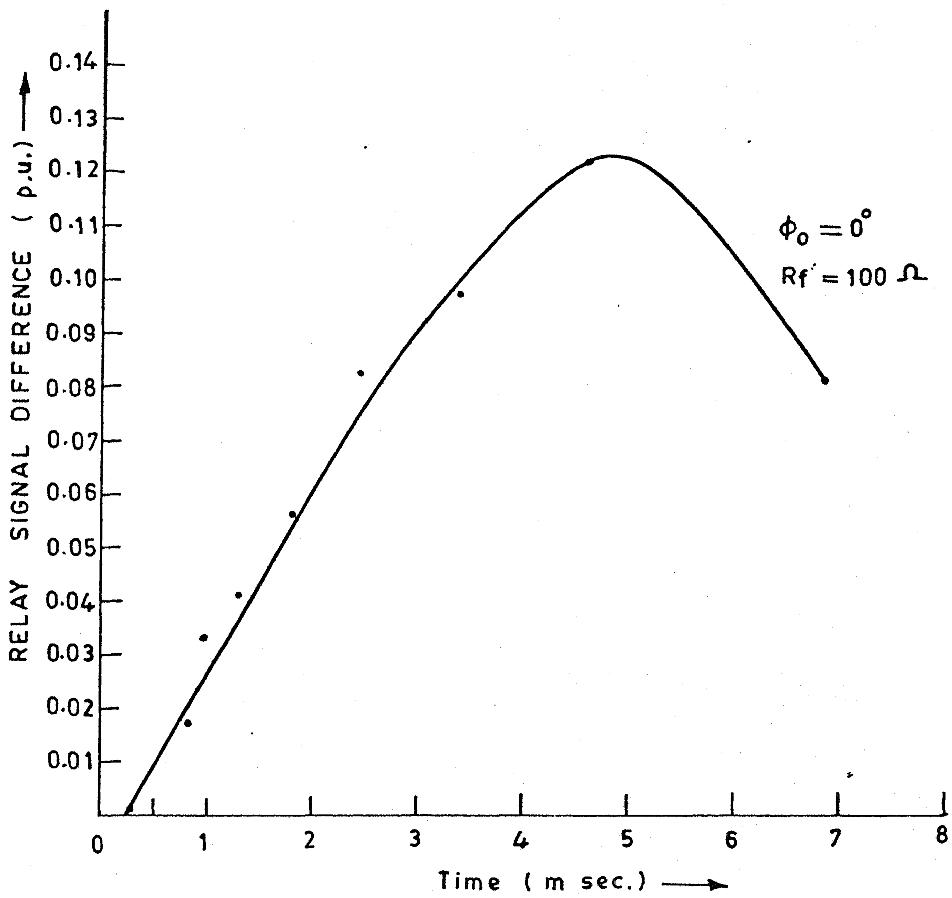


FIG. 11 Relay signal difference $|S_1| - |S_2|$ for internal fault on sample network.

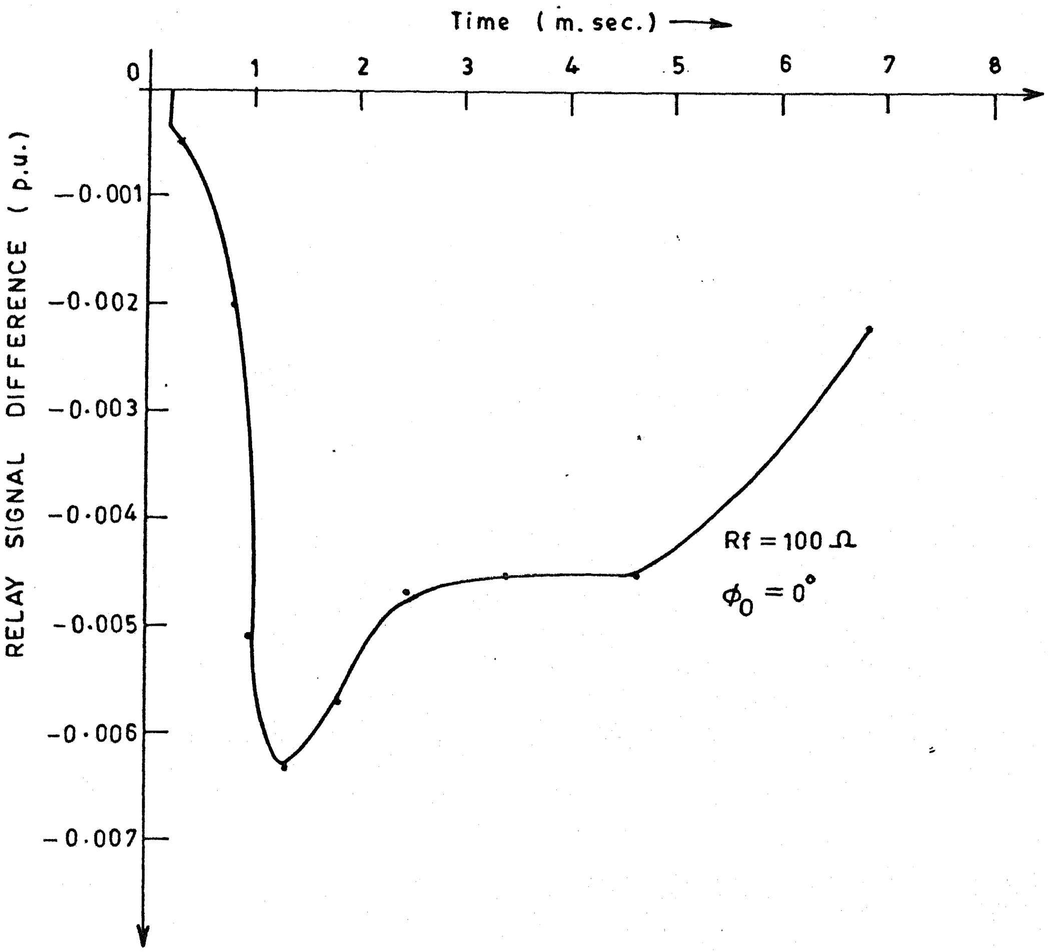


FIG. 12 RELAY SIGNAL DIFFERENCE FOR EXTERNAL FAULT IN THE SAMPLE POWER SYSTEM NETWORK.

found using the symmetrical component transformation matrix [T].

$R_{set}^{(m)}$ is the surge impedance for the m^{th} mode. The amplitude comparison scheme is applied to each signal pair. The tripping or blocking signal is given by the first positive outcome of the comparison, i.e.

$$|S_1^{(m)}| > |S_2^{(m)}| \qquad m = 0,1,2,3,4,5 \; : \text{INTERNAL FAULT}$$

$$|S_1^{(m)}| < |S_2^{(m)}| \qquad : \text{EXTERNAL FAULT} \tag{31}$$

3.4 TESTING ON A SAMPLE POWER SYSTEM

The above scheme is tested by simulating fault conditions on the sample powers system of Chapter 2. The signals S_1 and S_2 are shown in Figs. 11,12 for internal and external faults. The values of signal difference confirm the above principles.

3.5 IMPLEMENTATION OF THE RELAYING SCHEME

With the emergence of fast and reliable microprocessors and dedicated software, digital protection of transmission lines has become feasible. Digital protection offers the two fold advantage of speed and flexibility.

Digital travelling wave algorithms use the post fault transient current and voltage waveforms

which are obtained after filtering out the prefault sinusoidal voltage and current using a simple digital filter. The relaying decisions are thus based on the fault generated components of voltage and current. Very few algorithms based on travelling wave phenomenon have been proposed. This section suggests the algorithm for implementing the relaying scheme proposed in the earlier sections.

3.5.1 Fault Detection Algorithm

This is similar to the algorithm proposed by Mann and Morrison [12]. One counter is provided for each phase voltage. The difference between currently sampled instantaneous voltage of each phase and the corresponding value of the previous cycle is stored. If this difference exceeds a pre set value (0.05 pu) in any case the counter of that phase is incremented by one. If not, the counter is decremented by one. Once the counter reaches a pre set value in any of the six phases, fault is assumed to have occurred and the relaying algorithm is executed.

3.5.2 Relaying Algorithm

Once the fault detection algorithm yields a picture outcome, the relaying algorithm is executed. This is as follows :-

Step 1: Calculate by cycle to cycle comparison, the fault generated components of all line currents of five sample sets backwards from the set at which fault detection algorithm gave a logical yes output.

Step 2: Calculate the modal components of phase voltages and line currents. At the first instance, the first sample set is considered.

Step 3: Execute steps 4 to 5 for the modes 0,1,2,3,4,5.

Step 4: Check if $|S_1^{(m)}| > |S_2^{(m)}|$. If so, start the transmitter. If trip signal arrives from other end, give the trip signal to circuit breaker.

Step 5: If $|S_1^{(m)}| < |S_2^{(m)}|$, block the transmitter signal.

Step 6: Take the next sample set and go to step 2.

3.6 DISCUSSIONS AND CONCLUSION

The chapter presents a new relaying scheme for protection of multiphase transmission lines. The schemes use the presence of travelling wave component in the post fault waveforms to derive the appropriate relay input signals. The proposed scheme is simple and does not require complex computations.

The viability of the scheme has been demonstrated on a sample 3-bus power system network. The chapter also presents an algorithm for digital implementation of the scheme.

The relaying schemes based on travelling waves are immune to power swings since the latter are comparatively slow transients. The transients due to switching appears as an external disturbance and hence will not operate the relay. Since the fault detection takes place before the CT's saturate, the associated problems are eliminated. The lightning strokes have a high frequency component associated with them in the MHz range. These could be eliminated using a simple lowpass filter.

The fault detection algorithm proposed in this chapter can be used also for various analog travelling wave relaying algorithms for digital implementation.

TABLE 1

Relay Input Signals for an Internal Fault

Time	v_{fs}	i_{fs}	$\lvert S_1 \rvert$	$\lvert S_2 \rvert$
$t < \tau_1$	0	0	0	0
$t = \tau_1^-$	v_{ff}	$-v_{ff}/Z_o$	$2\lvert v_{ff} \rvert$	0
$t \geq \tau_1^+$ till another wave pair arrives	$(1+\rho_s)v_{ff}$	$-(1-\rho_s)v_{ff}/Z_o)$	$2\lvert v_{ff} \rvert$	$2\lvert \rho_s v_{ff} \rvert$

CHAPTER 4

CONCLUSION

The ever increasing demand for power have prompted power engineers to search for newer and more efficient methods of power generation and transmission. In this context, high phase order transmission (HPOT) system seems to be a promising way of transmitting greater amounts of power to the load centres at reasonable cost and at better voltage profile.

Any proposal for design of a multiphase power system must be accompanied by adequate measures against faults. The ultra high speed relaying schemes based on travelling wave phenomenon offer the dual advantage of speed and reliability. Hence, UHS protection is particularly suited to protection of EHV/UHV transmission lines. The use of such schemes is particularly facilitated by the advent of high speed, low cost microcomputers and microprocessors.

Accordingly, the thesis aims primarily at designing protective relaying schemes for six phase transmission lines based on travelling wave phenomenon. For this a frequency domain Pi-model of a six phase transmission line has been developed. The thesis has

also explored a novel technique for Laplace transform inversion which, it is hoped, shall facilitate transient studies on power systems.

Designing a relaying scheme for a multiphase transmission line requires that a model for the transmission line which would lend itself to easy transient analysis be developed first. To this end, a frequency (S-domain) model for the transmission line has been developed. The model has been developed keeping in mind, that it should be helpful in studying generation and propagation of travelling waves on the transmission line. Laplace domain techniques have been used for this purpose.

In the past, because of difficulties in inverting a Laplace domain function to the time domain, use of Laplace transform was somewhat restricted. The thesis has suggested a new method of inverting Laplace transforms. The technique does not require any complex operations like contour integration etc. and simply reduces the Laplace inversion to a matrix multiplication. The matrix in question, is a constant, and can be stored for repeated use. The thesis also presents a generalised nodal analysis for fault analysis of multiphase power systems. The S-domain bus admittance matrix has been used for the purpose. Similarly, the Pi-model of the transmission line is also suited to nodal analysis.

The viability of model developed has been demonstrated on a sample 3-bus system. The voltage waveforms for different values of fault resistance and fault initiation angle ϕ_o have been shown.

Ultra high speed clearing of faults improves the transient stability of the systems. The fault clearance time depends on both the relay speed as well as delay associated with the circuit breaker. With development of ultra high speed circuit breaker, ultra high speed relaying schemes have gained importance. But very few relaying schemes based on travelling wave phenomenon has been proposed so far. This paper presents an amplitude comparison relaying scheme based on travelling wave phenomenon along with fault detection algorithm.

The amplitude comparison scheme decomposes the fault generated voltages and currents to their modal components and makes trip/block decisions based on these modal components. Tripping is initiated if the fault is detected as a forward fault at both ends. The underlying principles have been verified by fault simulation on a sample power system. The effect of fault resistance and fault initiation angle has been included in the study. With the use of modal components, all types of faults can be protected against, using the

same algorithm. Hence the relaying scheme can be considered to be a new type of polyphase relay.

Digital protection of transmission line has largely been restricted to various types of distance relay algorithms which involve the computation of power frequency impedance from the complex post fault waveforms of voltages and currents. Different types of analog and digital filters have been constructed for extractingthe fundamental frequency components of voltages and currents. However, the accuracy of determining this component depends on the extent of filtering. Further, the nature of filtering depends upon the rigour of modelling the line. Either more elaborate modelling or complex filtering can result in inordinate time delays. But travelling wave relays require that only the prefault component of voltages and currents be filtered out. Further, only a few samples are required for this purpose. Hence, the speed of operation of UHS relays is much faster.

The prefault voltages have been filtered out using the cycle to cycle comparison method proposed by Mann and Morrison. The viability of these algorithms has been tested on Dec 1090.

Future Scope of Work : The modelling of transmission line, fault simulation and testing have been done using

software techniques. Such techniques, while demonstrating the theoretical feasibility of schemes, do not bring forward the practical difficulties encountered during hardware, on-line implementation.

Moreover, the thesis has considered only simple multiphase systems. Future work could include the modelling of transformers, voltage and current transducers, generators in the overall system to obtain a more realistic picture.

Lastly, relaying principles developed shall need reassessment when applied to double circuit lines, series and shunt compensated lines and multi-terminal lines.

REFERENCES

1. K.L. Hicks and W.L. Butt, 'Feasibility and Economics of UHS Fault Clearing', IEEE Trans. on PAS, Vol. PAS-99, Nov/Dec 1980; 2138-2145.

2. R.D. Berglund, W.A. Mittlestadt, M.L. Shelton, P. Barkan, C.C. Dewey and K.M. Skreiner, 'One-cycle fault interruption at 500 kV - System Benefits and Breaker Design', IEEE Trans. on PAS Vol. PAS-93, Sept/Oct 1974, 1240-1253.

3. A.T. Johns and R.K. Agarwal, 'Digital Simulation of Faulted EHV Transmission Lines with Particular Reference to VHS Protection' Proc. IEE Vol. 123 Apr. 1976; 353-359.

4. G.C. Kothari, K. Parthasarthy, B.S. Ashok Kumar and H.P. Khincha, 'Computer Aided Analysis of High Speed Protective Relays', Proc. IEE, Vol. 121 July 1974; 687-694.

5. W.D. Humpage, K.P. Wong, M.H. Al-Dabagh and E.S. Mukhtar, 'Dynamic Simulation of High Speed Protection', Proc. IEE, Vol. 121, June 1974; 474-480.

6. M. Vittins, 'A Correlation Method for Transmission Line Protection', IEEE Trans. on PAS, Vol. PAS-97, Sept/Oct 1978; 1607-1616.

7. M. Chamia and S. Liberman, 'UHS Relay for EHV/UHV Transmission Lines - Development, Design and Application', IEEE Trans. on PAS, Vol. PAS-97, Nov/Dec 1978; 2104-2116.

8. L. Matele, 'Development, Design, Application and Field Experience of an UHS Relaying System for EHV/UHV Transmission Lines', Conference Papers, Vol. 1; Conference on Power System Protection, Madras (INDIA), April 16-19, 1980.

9. A.T. Johns and R.K. Agarwal, 'New UHS Directional Blocking Scheme for Transmission Line Protection', Second International Conference on Developments in Power System Protection, London, Conference Publication No. 185, IEE, 1980; 141-145.

10. T. Takagi, J. Baba, U. Katushiko and T. Sakaguchi, 'Fault Protection based on Travelling Wave Theory - Part I Theory', Presented at IEEE PES Summer Meeting, 1977, Paper No. A77-750-3.

11. Y. Akimoto, T. Yamamoto, H. Hosawaka, T. Sakaguchi, T. Yoshido and S. Nishida, 'Fault Protection based on Travelling Wave Theory (Part I Theory)', Electrical Engineering in Japan, Vol. 98, Jan/Feb 1978; 79-86.

12. B.J. Mann and I.F. Morrison, 'Digital Calculation of Impedance for Transmission Line Protection', IEEE

13. B.J. Mann and I.F. Morrison, 'Relaying a Three Phase Transmission Line with a Digital Computer', IEEE Trans. on Power Apparatus and Systems, Vol. PAS-90, March/April 1971; 742-750.

14. M. Ramamoorthy and K.N. Verma, 'A Rigorous Method for Fault Current Evaluation - Application to Relaying', Proc., Symposium of EHV Engineering Techniques and Testing Equipment, University of Roorkee, Vol. I, Sept. 1980; 4-21 to 4-33.

15. T. Takagi and Y. Yamakosi, 'Digital Differential Relaying System for Transmission Line Primary Protection using Travelling Wave Theory - its Theory and Field Experience', Presented at the IEEE PES Winter Meeting, 1979, Paper No. A79-096-9.

16. S.N. Tiwari and L.P. Singh, 'Mathematical Modelling and Analysis of Multiphase Systems', IEEE Trans. on PAS-101, 1982; 1784-1793.

17. M.M. Chowdhary and L.P. Singh, 'Generalised Mathematical Modelling of n-phase Power Systems', Electric Machines and Power Systems, Vol. 10, No. 5,

18. J.P. Bickford, N. Mullineux and J.R. Reed, 'Computation of Power Systems Transients', IEE Monograph, No. 18, Peter Peregrinus Ltd., England, 1980.

19. Richard Bellman, R.E. Kalaba and J.E. Lockett, 'Numerical Inversion of Laplace Transform', American Elsevier Publishing Company, New York, 1966.

20. M.T. Yee and J. Eztergalyos, 'UHS Relay of EHV/UHV Transmission Lines - Installation, - staged fault tests and operational experience', IEEE Transactions on Power Apparatus and Systems, Vol. PAS-97, Sept/Oct 1978; 1814-1825.

21. K.V. Desicachar and L.P. Singh, 'Digital Travelling Wave Protection of Transmission Lines', Electric Power Systems Research, 7, 1984; 19-28.

APPENDIX - A

THE ELEMENTS OF THE INVERSE MATRIX WITH DIVISION BY WEIGHTS

In these tables, we present the inverse of the matrix $(w_i x_i^{j-1})$, i,j = i, 2.....,N, for N = 6.

N = 6

ROW 1	1.827E 1
	-2.263E 2
	9.730E 2
	-1.886E 3
	1.691E 3
	-5.702E 2
ROW 2	-5.213E 0
	1.881E 2
	-1.078E 3
	2.390E 3
	-2.309E 3
	8.160E 2
ROW 3	2.636E 0
	-1.038E 2
	8.347E 2
	-2.237E 3
	2.429E 3
	-9.275E 2
ROW 4	-1.620E 0
	6.546E 1
	-5.750E 2
	1.794E 3
	-2.208E 3
	9.275E 2
ROW 5	1.063E 0
	-4.337E 1
	3.943E 2
	-1.311E 3
	1.770E 3
	-8.160E 2
ROW 6	-6.387E -1
	2.616E 1
	-2.411E 2
	8.234E 2
	-1.159E 3
	5.702E 2

APPENDIX - B

NEGATIVES OF LOGARITHMS OF ROOTS OF SHIFTED LEGENDRE POLYNOMIALS

The negative logarithms tabulated are

$$t_i = -\ln x_i$$

where

$$P_N^*(x_i) = 0 \quad \text{for } i = 1,2,\ldots,N;\ N = 6$$

N = 6
3.388323
1.775520
0.965769
0.479150
0.185601
0.034348

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